Risk analysis of annuity conversion options in a stochastic mortality environment

Joint work with Alexander Kling and Jochen Russ

Katja Schilling
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Research Training Group 1100
Introduction

Model framework

Numerical results
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Numerical results
Unit-linked deferred annuities

▶ Premiums are accumulated in a fund
▶ At retirement, the policyholder has the choice between
  ▶ account value as a lump sum
  ▶ converting the account value into an annuity at then prevailing rates
▶ Resulting annuity payment highly depends on
  ▶ fund value
  ▶ interest rate expectations
  ▶ mortality expectations
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Annuity conversion options

- Insurance companies add guarantees to pure unit-linked deferred annuities, e.g.
  - Guaranteed annuity options (GAOs)
  - Guaranteed minimum income benefits (GMIBs)
- Such options can become unexpectedly valuable (cf. UK)
## Literature

- **Pricing GAOs under deterministic mortality**: e.g.
  - Boyle and Hardy (2003), Ballotta and Haberman (2003), Van Haastrecht et al. (2010)

- **Pricing GAOs under stochastic mortality**: e.g.

- **Pricing GMIBs**: e.g.
  - Bauer et al. (2008), Marshall et al. (2010), Bacinello et al. (2011)
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Research objectives

1. What **risk** do annuity conversion options imply for the insurer?
2. How does the risk change with different **option types**?
3. Is it possible to reduce the risk by applying **risk management strategies**?
4. What risk (**fund, interest rate or mortality risk**) dominates the total risk?
Notation

- $0$ to $T$: deferment period/retirement date
- $x$: policyholder’s age at inception of the contract ($t = 0$)
- $\tau_x$: remaining lifetime
- $P_0$: single premium
- $A_t$: account value
- $a_T$: value of an immediate annuity with unit amount per year
Option types

- **Guaranteed annuity option (GAO)**
  - certain minimum conversion rate \( g \)
  - for converting the account value into a lifelong annuity at time \( T \)
  - \( g \): annual annuity per unit account value at time \( T \)
  
  \[
  V_T^{GAO} = 1 \{ \tau_x > T \} \max \{ g A_T a_T - A_T, 0 \}
  \]

- **GAO with limit**

- **Guaranteed minimum income benefit (GMIB)**
Option types

- **Guaranteed annuity option (GAO)**
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  V_T^{GAO} = 1_{\{\tau_x > T\}} g A_T \max \left\{ a_T - \frac{1}{g}, 0 \right\}
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  V_{T}^{GAO} = \mathbb{1}_{\{\tau_x > T\}} gA_T \max \left\{ a_T - \frac{1}{g}, 0 \right\}
  \]

- **GAO with limit**
  - conversion rate \( g \) up to a maximum account value \( L \) (limit)
  
  \[
  V_{T}^{\text{Limit}} = \mathbb{1}_{\{\tau_x > T\}} g \min \{ A_T, L \} \max \left\{ a_T - \frac{1}{g}, 0 \right\}
  \]

- **Guaranteed minimum income benefit (GMIB)**
  - fixed minimum annuity amount \( M (= gG) \)
  
  \[
  V_{T}^{\text{GMIB}} = \mathbb{1}_{\{\tau_x > T\}} \max \{ gGa_T - A_T, 0 \}
  \]
## Risk management strategies

<table>
<thead>
<tr>
<th>No hedging</th>
<th>Hedging</th>
</tr>
</thead>
<tbody>
<tr>
<td>No option fee</td>
<td>A</td>
</tr>
<tr>
<td>Option fee</td>
<td>B</td>
</tr>
</tbody>
</table>

- **Strategy B**
  - Option fee is invested in money market instruments

- **Strategy C**
  - Static hedge against the financial risk during the deferment period

- Assumption: Option fee = Hedging costs under strategy C
We analyze...

... the insurer’s loss distribution at time $T$

- for each combination of option type and risk management strategy
- by performing a Monte Carlo simulation

Modeled risk processes

- **Fund value**: Geometric Brownian motion
  \[
  dS(t) = (\lambda_S + r(t))S(t)dt + \sigma_S S(t)dW^S(t), \ S(0) > 0.
  \]

- **Short rate**: one-factor Cox-Ingersoll-Ross model
  \[
  dr(t) = \kappa(\theta - r(t))dt + \sigma_r \sqrt{r(t)}dW^r(t), \ r(0) > 0.
  \]

- **Mortality**: 6-factor forward model (cf. Bauer et al., 2008a)
  \[
  d\mu(t, T, x) = \alpha(t, T, x)dt + \sigma(t, T, x)dW^\mu(t), \ \mu(0, T, x) > 0.
  \]
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Numerical results
Risk of GAOs seems to be quite low for the insurer
Insurer’s loss (base case) (2)

- GAO in-the-money ⇔ Limit in-the-money
Insurer’s loss (base case) (3)

- Risk of GMIBs seems to be much higher than risk of GAOs/Limits
Insurer’s loss (very low interest rates)

- Risk of GAOs is now the highest
- Limit becomes relevant
- Option values: 0.0157 ($\text{GAO}_A$), 0.0093 ($\text{Limit}_A$), 0.1386 ($\text{GMIB}_A$)
Insurer’s loss (base case) - Hedging strategies

- Strategy B: In many cases low profit, but risk is not significantly reduced
- Strategy C: Risk is significantly reduced
- Mortality is not negligible
Main risk driver

<table>
<thead>
<tr>
<th>Sensitivity with respect to...</th>
<th>Option most affected</th>
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<tr>
<td>Interest rate level $\theta$</td>
<td>GAO/Limit</td>
</tr>
<tr>
<td>Volatility of mortality $\sigma$</td>
<td>GAO/Limit</td>
</tr>
<tr>
<td>Fund volatility $\sigma_S$</td>
<td>GMIB</td>
</tr>
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- Interest rate risk and mortality risk seem to dominate GAO and Limit
- Fund risk seems to dominate GMIB
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- Interest rate risk and mortality risk seem to dominate GAO and Limit
- Fund risk seems to dominate GMIB

Question of decomposing the risk between the different risk drivers requires further research!
Thank you very much for your attention!
References (1)

References (2)

## Model parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>$x$</td>
<td>50</td>
</tr>
<tr>
<td>Term to maturity</td>
<td>$T$</td>
<td>15</td>
</tr>
<tr>
<td>Single premium</td>
<td>$P_0$</td>
<td>1</td>
</tr>
<tr>
<td>Conversion rate</td>
<td>$g$</td>
<td>0.05</td>
</tr>
<tr>
<td>Limit</td>
<td>$L$</td>
<td>1</td>
</tr>
<tr>
<td>Guaranteed account value</td>
<td>$G$</td>
<td>1</td>
</tr>
<tr>
<td>Number of realizations</td>
<td>$N$</td>
<td>10,000</td>
</tr>
<tr>
<td>Number of discretization steps</td>
<td>$n$</td>
<td>1,500</td>
</tr>
<tr>
<td>GBM initial value</td>
<td>$S(0)$</td>
<td>100</td>
</tr>
<tr>
<td>GBM risk premium</td>
<td>$\lambda_S$</td>
<td>0.03</td>
</tr>
<tr>
<td>GBM volatility</td>
<td>$\sigma_S$</td>
<td>0.22</td>
</tr>
<tr>
<td>CIR initial value</td>
<td>$r(0)$</td>
<td>0.0029</td>
</tr>
<tr>
<td>CIR speed of reversion</td>
<td>$\kappa$ ($\tilde{\kappa}$)</td>
<td>0.2 (0.2)</td>
</tr>
<tr>
<td>CIR mean level</td>
<td>$\theta$ ($\tilde{\theta}$)</td>
<td>0.045 (0.045)</td>
</tr>
<tr>
<td>CIR volatility</td>
<td>$\sigma_r$ ($\tilde{\sigma}_r$)</td>
<td>0.075 (0.075)</td>
</tr>
<tr>
<td>Explicit GBM-CIR correlation</td>
<td>$\rho$</td>
<td>0</td>
</tr>
<tr>
<td>Limiting age</td>
<td>$\omega$</td>
<td>121</td>
</tr>
</tbody>
</table>