Pricing of Debt and Loan Guarantees using Stochastic Delay Differential Equations

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- Antoine Tambue (Bergen, Norway)
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1. Introduction
   - Motivation

2. Evaluation of Corporate Claim in a Single Class of Debt
   - Derivation of RPDE

3. Evaluation of Corporate Claims
   - Evaluation of Debt in a Levered Firm
   - Evaluation of Loan Guarantees

4. Testing using Real Data
   - Numerical Scheme
   - Testing

5. Conclusion

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Pricing of Debt and Loan Guarantees with SDDEs
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5. **Conclusion**
In ([Merton, 1974]), the author derived a partial differential equation such that any claim whose value depends on firm value and time, is a solution under some suitable boundary and final conditions.

- Merton used a constant volatility and a firm value without time delay.
- We extend the work of Merton to a non constant volatility and include time delay in the firm value.
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We extend the work of Merton to a non constant volatility and include time delay in the firm value.
Several authors show that the past dependence of the stock price process is an important feature to capture the better prediction of the future dynamic (see [Chang and Youree, 1999, Mohammed et al., 2007]). This is a motivation for introducing delay models in corporate finance pricing.
Because of the isomorphic relationship between a levered equity and a European call option (see Merton [Merton, 1973]) on the one hand, and the isomorphic correspondence between loan guarantees and common stock put options (see Merton [Merton, 1977]), we can claim that results obtained in the theory of option pricing are feasible in corporate liabilities pricing.
In this work, we derive a formula for the price of an option used for the pricing of corporate defaultable bonds and adopt this approach to evaluate loan guarantees for companies in financial distress. We provide the implicit Euler-Maruyama scheme to approximate the company value and test our model against real data. We also compare our simulations with Merton’s model.
Some standard definitions in Finance

- **Corporate Claim**: An official request for money usually in the form of compensation, from a corporation.
- **Contingent Claim**: A claim that may or may not occur, but for which provision is made in a company’s accounts.
- **Debt Security**: A security issued by a company or government which represents money borrowed from the security’s purchaser and which must be repaid at a specified maturity date, usually at a specified interest rate.
- **Defaultable Bond**: A financial security which promises to pay $1 at maturity time $T$ if no default occurs prior to $T$.
- **Loan Guarantees**: Loan on which a promise is made by a third party or guarantor that he or she will be liable if the creditor fails to fulfill their contractual obligations.
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**Definition 1**

(European Call (resp. put) Option, Strike Price)

A *European call (resp. put) option* is an option that gives its holder the right (but not the obligation) to purchase (resp. sell) a specified number of shares of the underlying asset at an agreed-upon price (*strike or exercise price*) on the expiration date of the contract, regardless of the prevailing market price of the underlying asset.
We will assume the following:

1. the value of the company is unaffected by how it is financed (the capital structure irrelevance principle).

2. the value $V(t)$ of the firm at time $t$, follows the nonlinear stochastic delay differential equation:

$$
\begin{align*}
    dV(t) &= (\alpha V(t) V(t - L_1) - C)dt \\
    &+ g(V(t - L_2)) V(t) dW(t), \quad t \in [0, T] \\
    V(t) &= \varphi(t), \quad t \in [-L, 0],
\end{align*}
$$

(1)
where $\alpha$ is the constant riskless interest rate of return on the firm per unit time, $C$ is the total amount payout by the firm per unit time to either the shareholders or claims-holders (e.g., dividends or interest payments) if positive, and it is the net amount received by the firm from new financing if negative; $g : \mathbb{R} \to \mathbb{R}$ is a continuous function representing the volatility function on the firm value per unit time; the initial process $\varphi : \Omega \to \mathcal{C}([L, 0], \mathbb{R})$ is $\mathcal{F}_0$-measurable with respect to the Borel $\sigma$-algebra of $\mathcal{C}([L, 0], \mathbb{R})$, where $L = \max(L_1, L_2)$ is a positive constant; the process $W$ is a one dimensional standard Brownian motion adapted to the filtration $(\mathcal{F}_t)_{0 \leq t \leq T}$. 

\[ \text{where } \alpha \text{ is the constant riskless interest rate of return on the firm per unit time, } C \text{ is the total amount payout by the firm per unit time to either the shareholders or claims-holders (e.g., dividends or interest payments) if positive, and it is the net amount received by the firm from new financing if negative; } g : \mathbb{R} \to \mathbb{R} \text{ is a continuous function representing the volatility function on the firm value per unit time; the initial process } \varphi : \Omega \to \mathcal{C}([L, 0], \mathbb{R}) \text{ is } \mathcal{F}_0\text{-measurable with respect to the Borel } \sigma\text{-algebra of } \mathcal{C}([L, 0], \mathbb{R}), \text{ where } L = \max(L_1, L_2) \text{ is a positive constant; the process } W \text{ is a one dimensional standard Brownian motion adapted to the filtration } (\mathcal{F}_t)_{0 \leq t \leq T}. \]
Assume there exists a claim with market value, \( Y(t) \), at any point \( t \), where \( Y(t) = f(V(t), t) \) follows the dynamics of this claim’s value in stochastic differential equation form as below

\[
\begin{align*}
  dY(t) &= (\alpha_y Y(t) - C_y)dt \\
  &+ g_y(Y(t - L_2))Y(t)dW_y(t), \quad t \in [0, T] \\
  Y(t) &= \varphi_y(t), \quad t \in [-L, 0],
\end{align*}
\]

where \( \alpha_y \) is the constant riskless interest rate of return per unit time on this claim; \( C_y \) is the amount of payout per unit time on this claim; \( g_y : \mathbb{R} \to \mathbb{R} \) is a continuous function representing the volatility function of the return on this claim per unit time; the initial process \( \varphi_y : \Omega \to C([-L, 0], \mathbb{R}) \) is \( \mathcal{F}_0 \)-measurable with respect to the Borel \( \sigma \)-algebra of \( C([-L, 0], \mathbb{R}) \), where \( L = \max(L_1, L_2) \) is a positive constant;
the process $W_y$ is a one dimensional standard Brownian motion adapted to the filtration $(\mathcal{F}_t)_{0 \leq t \leq T}$. Assume that the debt accumulates interest compounded continuously at a rate of $r$, that is $B(t) = B(0)e^{rt}$.

Assume that $Y(t)$ can be replicated using self-financed strategy.

Then a random partial differential equation (RPDE) for $f$ is given by

$$\frac{1}{2}g^2(V(t - L_2))v^2 f_{vv} + (rv - C)f_v + f_t - rf + C_y = 0, \quad 0 < t < T$$

For any claim whose value depends on the value of the firm and time, the above equation must be satisfied under some specific final and boundary conditions.
To see this:

For a given \( Y(t) = f(V(t), t) \), the \( \alpha_y, g_y, dW_y \) and the corresponding \( \alpha, g, dW \) in SDDE (1) are similar. Applying Ito formula to \( Y(t) = f(V(t), t) \), we have the equality almost surely of the coefficients of the corresponding terms \( dt \) and \( dW(t) \) in the SDDEs for \( Y(t) \).

Since the return on the portfolio is non stochastic and there is no arbitrage condition, by self-financing and replication strategy, and assuming the total investment in the portfolio is zero, we obtain the RPDE (3).
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Assumptions

Assume the company is financed by a single class of debt and the equity. Furthermore, assume the following agreements are made in the contract according to the bond issue:

1. the firm must pay an amount $B(T)$ to the debtholders at the maturity date $T$;

2. in case the firm cannot make the payment, the debtholders take over the company and the equityholders lose their investment;

3. the firm is not allowed neither to issue a new senior claim on the firm nor to pay cash dividend during the option life.
Assumptions

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Solution of the RPDE

By assumption 3., there are no coupon payments, therefore we can set \( C = C_y = 0 \) in equation (1) and (2) respectively. Equation (3) becomes

\[
\frac{1}{2} g^2 (V(t - L_2)) v^2 f_{vv} + rvf_v + f_t - rf = 0, \quad 0 < t < T
\]  

(4)
Proposition 1

If $f$ is solution to the parabolic RPDE (4) with final and boundary conditions

$$f(v, T) = \max(v - B(T), 0), \; v > 0$$  \hspace{1cm} (5)

$$f(0, t) = 0, \; f(v, t) \sim v - B(T)e^{-r(T-t)} \text{ as } v \to \infty,$$  \hspace{1cm} (6)

then the equity value of the company is given by

$$f(V(t), t) = V(t)\Phi(x_1) - Be^{-r(T-t)}\Phi(x_2), \; t \in [T - l, T]$$  \hspace{1cm} (7)
Proposition 2 (continued)

where \( l = \min[L_1, L_2] \),

\[
\Phi(x) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy,
\]

\[
x_1 = \log \frac{v}{B} + r(T - t) + \frac{1}{2} \int_{t}^{T} g^2(V(s - L_2)) ds \frac{\sqrt{\int_{t}^{T} g^2(V(s - L_2)) ds}}{\sqrt{\int_{t}^{T} g^2(V(s - L_2)) ds}}
\]

and \( x_2 = x_1 - \sqrt{\int_{t}^{T} g^2(V(s - L_2)) ds} \).
Theorem 3

Assume a levered firm is under the agreements 1., 2., 3. mentioned earlier. Moreover assume the following:

- the value \( V(t) \) of the firm at any time \( t \) can be written as
  \[
  V(t) = F(V(t), t) + f(V(t), t),
  \]
  where \( f(V(t), t) \) is the value of the equity, \( F(V(t), t) \) the value of debt at any time \( t \) before the maturity.

- the boundary condition and the final condition for the debt at \( t = T \) given as
  \[
  F(0, t) = f(0, t) = 0 \quad \text{and} \quad F(V(T), T) = \min[V(T), B].
  \]
Theorem 3

Assume a levered firm is under the agreements 1., 2., 3. mentioned earlier. Moreover assume the following:

- the value $V(t)$ of the firm at any time $t$ can be written as

$$V(t) = F(V(t), t) + f(V(t), t), \quad (8)$$

where $f(V(t), t)$ is the value of the equity, $F(V(t), t)$ the value of debt at any time $t$ before the maturity.

- the boundary condition and the final condition for the debt at $t = T$ given as

$$F(0, t) = f(0, t) = 0 \quad \text{and} \quad F(V(T), T) = \min[V(T), B]. \quad (9)$$
Theorem 4 (Continued)

Then the solution of equation (4) for $F$ under boundary and final conditions (9) is given by

$$
F(V(t), t) = Be^{-r(T-t)} \left\{ \Phi [N_2] + \frac{1}{d} \Phi [N_1] \right\}, \quad (10)
$$

where

$$
d \equiv \frac{Be^{-r\tau}}{V(t)}, \quad N_1 = -\left( \frac{1}{2} \int_t^T g^2(V(s - L_2))ds - \log(d) \right) \sqrt{\int_t^T g^2(V(s - L_2))ds}
$$

and

$$
N_2 = -\left( \frac{1}{2} \int_t^T g^2(V(s - L_2))ds + \log(d) \right) \sqrt{\int_t^T g^2(V(s - L_2))ds}.
$$
Proof.

The RPDE (4) can be written in term of $F$

$$\frac{1}{2} g^2 (V(t - L_2)) v^2 F_{vv} + rvF_v + F_t - rF = 0. \quad (11)$$

From relation (8) we have $F(V(t), t) = V(t) - f(V(t), t)$, and substitute for $F$ in (11) to get the RPDE (4) for $f$. Combining equations (8) and (9) we obtain the final condition

$$f(V(T), T) = \max[V(T) - B, 0]$$
By Proposition 1,

\[ f(V(t), t) = V(t)\Phi(x_1) - Be^{-r(T-t)}\Phi(x_2), \quad t \in [T - l, T] \quad (12) \]

where

\[ \Phi(x) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy, \]

\[ x_1 = \log \left( \frac{V(t)}{B} \right) + r(T - t) + \frac{1}{2} \int_t^T g^2(V(s - L_2)) ds \quad \sqrt{\int_t^T g^2(V(s - L_2)) ds} \quad (13) \]

and

\[ x_2 = x_1 - \sqrt{\int_t^T g^2(V(s - L_2)) ds}. \quad (14) \]
Continued.

From equation (8) and (12), we can write the value of the debt issue as

$$F(V(t), t) = Be^{-r(T-t)} \left\{ \Phi[h_2] + \frac{1}{d} \Phi[h_1] \right\}, \quad t \in [T - l, T]$$ \hspace{1cm} (15) \hspace{1cm} (15)

where

$$d \equiv \frac{Be^{-r(T-t)}}{V(t)} , \quad h_1 \equiv -\left( \frac{1}{2} \int_t^T g^2(V(s - L_2))ds - \log(d) \right) \frac{\sqrt{\int_t^T g^2(V(s - L_2))ds}}{\sqrt{\int_t^T g^2(V(s - L_2))ds}}$$

and

$$h_2 \equiv -\left( \frac{1}{2} \int_t^T g^2(V(s - L_2))ds + \log(d) \right) \frac{\sqrt{\int_t^T g^2(V(s - L_2))ds}}{\sqrt{\int_t^T g^2(V(s - L_2))ds}}.$$
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Assume the company is financed by a single class of loan guarantees and the equity. Furthermore, assume the following agreements are stipulated in the contract according to the loan guarantees issue:

1. in case the management on the maturity date is unable to make the payment promised, the guarantor will meet these payments with no uncertainty;

2. the firm is expected to pay an amount at least equal to its actuarial cost for the guarantee, so that in case this happens, the firm is required to default all its assets to the guarantor;

3. the firm is not allow neither to issue a new senior claim on the firm nor to pay cash dividend during the option life.
Assume the company is financed by a single class of loan guarantees and the equity. Furthermore, assume the following agreements are stipulated in the contract according to the loan guarantees issue:

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3. the firm is not allow neither to issue a new senior claim on the firm nor to pay cash dividend during the option life.
Theorem 5

Assume a levered firm are under the agreements 1., 2., 3. above. Moreover, assume the following conditions:

- the boundary condition

\[ G(0, t) = 0, \quad \text{(16)} \]

- the final condition for the loan guarantees at \( t = T \) given as

\[ G(V(T), T) = \max[B - V(T), 0]. \quad \text{(17)} \]

Then loan guarantees is given by

\[ G(V(t), t) = Be^{-r(T-t)} \Phi [d_1] + V(t) \Phi [d_2], \quad t \in [T - l, T]. \quad \text{(18)} \]
Theorem 5

Assume a levered firm are under the agreements 1., 2., 3. above. Moreover, assume the following conditions:

- **the boundary condition**
  \[
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Then loan guarantees is given by

\[
G(V(t), t) = Be^{-r(T-t)} \Phi [d_1] + V(t) \Phi [d_2], \quad t \in [T - l, T]. \quad (18)
\]
The data on stock returns are from the Center for Research in Security Prices (CRSP) database: http://www.crsp.com/. To estimate the volatility function $g$, we use the quadratic or linear interpolation of the memory part of data $Sdret$. As we only have yearly data set, we use also the interpolation to increase the amount of data if needed as the numerical schemes usually need small time steps to ensure their stabilities.
Consider the SDDE (1) in the time interval \([0, T]\), where the value of \(T\) is 10 and the value of the delay \(L_1 = L_2\) is 10. The time unit being the year. Moreover, we consider the values of \(\alpha\) and \(C\) are time dependent functions, which are constant within the year interval. We solve numerically equation (1) by using the \(\theta\)-semi implicit Euler-Maruyama scheme by

\[
V_{n+1} = V_n + \Delta T \left[ \theta (\alpha_{n+1} V_{n+1} V_{n-m+1} - C_{n+1}) + (1 - \theta)(\alpha_n V_n V_{n-m} - C_n) \right] + g(V_{n-m}) V_n \Delta W_n \quad n = 1 \ldots M, \quad 0 \leq \theta \leq 1, \quad L = m\Delta T, \quad (19)
\]
where $\Delta T = T/M$ is the time step size, $M$ the total number of time subdivisions, $V_n$ is the approximation of $V(t_n)$, $t_n = n\Delta T$, $\alpha_n = \alpha(t_n)$, $C_n = C(t_n)$, and $\Delta W_n = W(t_{n+1}) - W(t_n)$ are standard Brownian increments, independent identically distributed $\sqrt{\Delta T} N(0, 1)$ random variables.
To approximate the expected value of the solution $V$, we use Monte Carlo to compute the mean of the numerical solution sample from (19). In the simulations, we test the delay model and Merton’s model against real data for the following the companies: The companies we use in our simulation are those with $C_y$ in their data set, namely:

A  Great Northern Iron Ore Pptys (?? and ??),
B  Magna International Inc (3 and ??).
Figure: Firm A (Great Northern Iron Ore Pptys): Merton's, $\mathbb{E}(\Delta t)_t$.
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Figure: Firm A (Great N. I. O. Pptys): Delay model, $T = 5$, $L = 10$, $g =$ linear interpolation of the standard deviation of daily returns $S_d$. 

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Figure: Firm B (Magna Int Inc): Merton's, $T = L = 10$, $g =$ mean of the standard deviation of daily returns.
Figure: Firm B (Magna Int Inc): Delay model, $T = L = 10$, $g = \text{linear interpolation of the standard deviation of daily returns } S_dret$. 

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Pricing of Debt and Loan Guarantees with SDDEs
We suggest a delay model for pricing corporate liabilities. Using replication and self-financed strategy we have derived a RPDE which includes non constant volatility and time delay in the firm value. We solve the RPDE and provide a formula for the price of equity value, debt value and loan guarantees considering a delay on the firm value.

We obtain numerical approximations for the company value using the implicit Euler-Maruyama scheme. We use our approximation scheme to test our model against real market data. Finally, we compare the delay model on the company value with Merton’s model. It turns out that our model so far gives better prediction of the future price of the value of the company.
THANK YOU FOR YOUR ATTENTION AND TIME
M. Arriojas, Y. Hu, S. Mohammed and G. Pap
A Delayed Black and Scholes Formula,

R. C. Merton
Theory of Rational Option Pricing,
For Further Reading II

F. Black and M. Scholes
The Pricing of Options and Corporate Liabilities.

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For Further Reading III

- R. J. Elliott and P. E. Kopp

- D. Lamberton, B. Lapeyre, N. Rabeau, and F. Mantion, editors
  Introduction to Stochastic Calculus Applied to Finance.
  *Stochastic Analysis*, Chapman and Hall/CRC Press,
R. C. Merton
On the Pricing of Corporate Debt: The Risk Structure of Interest Rates.

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An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees.
For Further Reading V

- **R. C. Merton**
  Continuous-Time Speculative Processes: Appendix to Paul A. Samuelson’s ’Mathematics of Speculative Price’.

- **M. Musiela and M. Rutkowski**
  Martingale Methods in Financial Modelling.

- **T. Zastawniak and M. Capiński**