GUESS: Problem Solving Method

Academic Learning Centre

University of Manitoba

TRADITIONAL TERRITORIES ACKNOWLEDGEMENT

The University of Manitoba campuses are located on original lands of Anishinaabeg, Cree, Oji-Cree, Dakota, and Dene peoples, and on the homeland of the Métis Nation.

We respect the Treaties that were made on these territories, we acknowledge the harms and mistakes of the past, and we dedicate ourselves to move forward in partnership with Indigenous communities in a spirit of reconciliation and collaboration.

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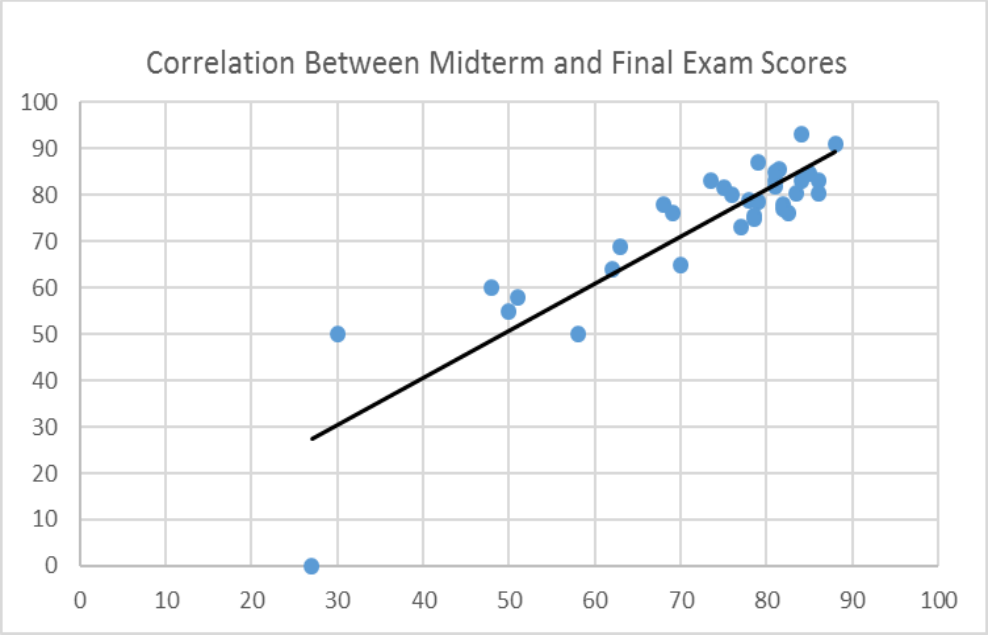
Sample Statistics Question



Given
Unknown
Equations
Set-up
Solve

QUESTION: The correlation between student midterm scores and final exam scores is 0.55. The midterm scores for all students has a mean of 85 and a standard deviation of 6, while the final exam has a mean of 70 and standard deviation of 9. Gloria scored 93 on the midterm. **What mark can we predict for her final exam score?**

The correlation between student midterm scores and final exam scores is 0.55. The midterm scores for all students has a mean of 85 and a standard deviation of 6, while the final exam scores has a mean of 70 and standard deviation of 9. Gloria scored 93 on the midterm. **What can we predict her final exam score to be?**

Given:	Correlation (r) = 0.55 Midterm mean ($\bar{x}$) = 85 Midterm Standard deviation (S_x) = 6 Final exam mean ($\bar{y}$) = 70 Final exam Standard deviation (S_y) = 9 Gloria's Midterm score (x) = 93

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$$\hat{Y} = b_0 + b_1 x$$

Unknown:	Intercept (b_0) = ? Slope (b_1) = ? Gloria's Final exam score ($\hat{Y}$) = ?

Regression

Linear Regression Model

Population Version:

Mean: $\mu_Y(x) = E(Y) = \beta_0 + \beta_1 x$

Individual: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

where ε_i is $N(0, \sigma)$

Sample Version:

Mean: $\hat{y} = b_0 + b_1 x$

Individual: $y_i = b_0 + b_1 x_i + e_i$

Parameter Estimators

$$b_1 = \frac{S_{XY}}{S_{XX}} = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2} = \frac{\sum (x-\bar{x})y}{\sum (x-\bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

Residuals

$e = y - \hat{y}$ = observed y – predicted y

Correlation and its square

$$r = \frac{S_{XY}}{\sqrt{S_{XX} S_{YY}}}$$

Standard Error of the Sample Slope

$$\text{s.e.}(b_1) = \frac{s}{\sqrt{S_{XX}}} = \frac{s}{\sqrt{\sum (x-\bar{x})^2}}$$

Confidence Interval for β_1

$b_1 \pm t^* \text{s.e.}(b_1)$ df $= n-2$

t-Test for β_1

To test $H_0 : \beta_1 = 0$

$$t = \frac{b_1 - 0}{\text{s.e.}(b_1)}$$ df $= n-2$

or $F = \frac{MSREG}{MSE}$ df $= 1, n-2$

Confidence Interval for the Mean Response

$\hat{y} \pm t^* \text{s.e.}(\text{fit})$ df $= n-2$

where $\text{s.e.}(\text{fit}) = s\sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{S_{XX}}}$

Prediction Interval for an Individual Response

$\hat{y} \pm t^* \text{s.e.}(\text{pred})$ df $= n-2$

where $\text{s.e.}(\text{pred}) = \sqrt{s^2 + (\text{s.e.}(\text{fit}))^2}$

Standard Error of the Sample Intercept

$$\text{s.e.}(b_0) = s\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{XX}}}$$

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Equations:	$b_1 = r * \frac{s_y}{s_x}$ $b_0 = \bar{y} - b_1\bar{x}$ $\hat{Y} = b_0 + b_1 x$

Set-up:	$b_1 = 0.55 * \frac{9}{6}$ **$b_1 = 0.825$** $b_0 = 70 – 0.825*(85)$ **$b_0 = -0.125$**

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Solve:	$\hat{Y} = b_1x + b_0$ $\hat{Y} = 0.825* 93 + (-0.125)$ $\hat{Y} = 76.6$

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