Mathematizing

Every day strategies in problem solving

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Introduction

A great deal of the history of research on culture and cognition has been dominated by the debate between those who sustain that there are no fundamental differences among human thought throughout the various cultures, and those who insist that they exist and that such differences are critical for the understanding of human nature. Based on the latter theory, the study's approach I am writing is about the way adults solve problems as individuals and as a group on specific time and place. This approach aims particularly to the study of problem-solving strategies on mathematical situations in everyday life followed by adults.

Learning is a social process by which human beings construct mathematical knowledge and skills in a co-operative way. There are learning opportunities within social interaction through co-operative dialogue, explanation and justification, as well as by the negotiation of meaning, not only at school but mostly in our daily jobs.

To understand our cognitive strategies as adults we need to conceived them in the context of our personal and occupational aims, tasks, purposes, salary or income, people we work with, the way we are organised and, the work tools and symbolic mediators of all the actions that conforms our activity.

Adults in conjunction with each other and interactively produce certain regularities and norms to speak and act in a mathematical way. As adults we mathematize our daily life, the mathematical functions in our social practice of jointly and interactively learning and teaching e.g. in the classroom, in our everyday

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activity at home, the grocery shopping, at work, even when we play. We mathematize every day life situations which can be potentially solved by means of mathematical operations, mediators and representations. The mathematizing is a culturally and mediated activity.

Knowledge cannot be reduced to a set of necessary tools and skills for work and life, transmitted by schooling systems (Cole M. & Scribner Sylvia, 1973). A broader view of knowledge refers to the processes by which people, regardless of age, gender, instruction, socio-economical level or culture, construct explanations, interpretations, plans and meanings.

Standing from an action theory point of view (Scribner Sylvia, 1997a,b,c) the issue between theoretical and practical does not include any practical assumption. Practical thinking, as used here, refers to all the thinking that is part of a larger set of activities and that seeks to achieve the goals this activities require. Goals may involve mental achievements (e.g. pricing customer deliveries) as well as manual achievements (e.g. loading a truck).

Cognition is considered for the purposes of this research as the unfolding of mind, culture, emotions, and actions in the socially organised activities in which adults are involved. Thus activities are conceived as a sequence of actions intellectually organised to achieve a specific goal, aim or purpose. Mental functioning involves individual organisation as well as social knowledge, both individual and social mental operations, internal operations and their expression in external operations; it also involves tools, symbolic systems, reasons and motives of a person, a group or a community historically and socially situated, all this in specific context.

In this paper I try to understand the way a group of painters in the house building industry in Mexico City generate and use some cognitive strategies to solve the problems they have to deal with in every day work. I aim to understand the way in which adults create and develop problem solving strategies for their practice, the way they mathematise collectively and individually.

Since Mexican economic crisis of 1982, be say millions of Mexicans had moved to Mexico City and other Mexican big cities as Monterrey, Guadalajara, Tijuana, Nogales, Ciudad Juárez and, to the United States looking for a better life. For a long time, it has been said that rural workers go through Mexico City just in their way to California, Texas, Illinois o wherever they can find a better paid job. But reality shows us that they do not, and this has been confirmed in the study of Carmen Bueno (1994). Instead, a lot people stay in Mexico City and live in the outer of the city where it has been a long battle to get services like water, streets, electricity. A lot of these peasants are hired in the construction industry.

Related to the migration phenomena is their poverty and marginalization. Newcomers have none or few schooling, the illiteracy rates are very high, among 6 million Mexican adults don't know how to read and write and another 12 million don't do it efficiently (Aguayo, S., 2000; Latapí, P. 1996). The education problem increases when we consider the number of workers that are not qualified for their jobs, and they have to learn while they are hired at the lowest in the hierarchical of workers in the construction industry as "peones" or unskilled labourers or "albañiles" or bricklayers (Bueno, Carmen 1994).

Since just a few adult apply to basic education programs at the National Institute for the Education of Adults or leave them incomplete and, also since certain adult education and training programs do not fit neither their educational needs of basic learning skills and knowledge for work and personal and family life; I see the need to study the way adults practical thought develops at work.

Problem-solving strategies

Problem-solving strategies are the actions and the use of conventional and personal collective symbolic tools for the solution of a problem. Hereafter when I refer to problem-solving, I mean only those which may be resolved through mathematical operations and representations. Such actions require the aid of instruments that permit and grant its adequate execution. Symbolic tools offer the power and guarantee to solve a problem effectively through mathematics. There are different kinds and manifestations of symbolic tools; like measurement systems, diagrams and drawings, counting and estimating, writing, etc.

Problem solving strategies are closely linked with our activities and with the practice, the activity and the occupation we are working on. It is considered that problem solving is a cognitive activity that appears as the result of the synthesis of external and internal operations and of collective and individual operations whose purpose is to achieve a goal or objective.

Therefore, the unit of analysis is the activity as a distinguished form of doing something, as an occupation, as a practice, so to be a teacher is a practice as well as medicine, engineer, a student, an accountant, a plumber, a blacksmith, a bricklayer or a swimmer. An activity is a practical system guided by aims, objectives, purposes that materialise through a set of concrete actions which are subordinated to the idea of an intentional and or conscious objectives. In a practice, actions are organised by a set of particular and discreet tasks in sufficient and needed interrelation for effectively work. The strategies adults create to deal with every day tasks of our activity develop from actions that are in intimate relation of order with the circumstances in which they occur, in accordance of the context. Then ,and at a second level of analysis this study refers with the unit of a goal-directed action. In the same sense, these strategies involve certain operations which imply the unconscious use of symbolic tolls, while action refer to the conscious aims and symbolic mediators. So operations refer to the third level of analysis to the routinely behaviours and accomplishments in an automatic way, but not necessarily conscious, operations are internalised actions in our intellectual and cognitive work.

Some researchers have centred their studies around situations on which people develop problem solving strategies that become routines (Lave, Jean 1991; Scribner Sylvia, 1997a, 1997b, 1997c), while others observed situations on which the solving of problems is adaptive (De la Rocha, Olivia 1985; Saxe, G. 1988). But on recent years, still other researchers, have demonstrated how both aspects may appear simultaneously or in a parallel form in certain kind of situations (Masingila, Joanna 1994; Milroy, Wendy 1992).

The study of mathematical practice in different cultures has been named ethnomathematics. Ubitaran D'Ambrosio (1985) a Brazilian mathematics teacher has introduced the concept of ethnomathematics and by it he refers to the influence of sociocultural factors in the teaching and learning of mathematics. The "ethno" prefix alludes to "identifiable cultural groups", such as tribal societies, work groups, children of a certain age, professional groups, etc. Scott, P. (1985:1-2) incorporated the meaning "ethno" into the mathematics content in his definition on ethnomathematics:

> Ethnomathematics lies at the confluence of mathematics and cultural anthropology. At one level, it is what might be called "math in the environment" or "math in the community". At another, related level, ethnomathematics is the particular (and perhaps peculiar) way of classifying, ordering, counting and measuring (Scott, P., 1985:2).

One of the most distinguished researchers in ethnomathematics is Allan Bishop (1988). He argues that there are six basic mathematical activities that are universal since they appear to occur in all the social groups he observed. This activities are necessary and sufficient for the development of mathematical knowledge. The activities he found were: counting, locating, measuring, designing, playing and explaining mathematics as a cultural knowledge that derives from a commitment of people in this universal activities in a stable and conscious way.

In a study on carpet and floor laying, Joanna Masingila (1994) proposes that estimators and carpet layers use not only mathematical concepts but several other mathematical procedures: measurement and problem solving.

Spatial visualisation plays an important role in the measurement process. It is used when a decision about how to cut a piece of carpet has to be done or in the elaboration of a mental image about how will the completed work look like.

Masingila observed many situations in which carpet layers measure in a non standardised way; this means that the procedure does not include the use of marked measuring instruments. Most of this situations involve an object to object measurement.

Workers use the mathematical process of problem solving when they have to make decisions over estimates and laying of carpets. The problems they encounter require different levels of skills for their solution. The more the shape of the measured space differs from the basic rectangle, the higher the level of skill required. Masingila observed four categories of strategies used by workers to face problems that arise during work: use of tools, use of an image, verification of possibilities, use of an algorithm:

When adults solve problems they create, use, acquire and develop: a) the procedures of taking into action and decide among and about different space configurations, b) different tools for the work and symbolic instruments or mediators, c) different kinds of images: pictures, designs, graphs, maps; d) a special way or mode to verify different possibilities of doing, representing, calculating and measuring; e) algorithms; and f) visualisations.

Methodological framework

The design of this research is qualitative with an ethnographic orientation.

Subjects

The painter's leader is Pablo, the first to move to Mexico City, who just went three years at elementary school and then decided to work as a fruit seller the same as his mother had taught him. He is a "wide open eyes" man, he says, for selling and making business with people.

Benito moved to Mexico City several years later than his stepbrother Pablo. Benito says "he is not good with people but he had good grades at school", so he continuing studying until he finish a technical high school and even tried several times to get in College because he wanted to be an agronomist engineer, and never could. Both, now are masters in painting, but the leader of the group is Pablo, he is the one that has more experience, the best painter and the one that has been longer in the business hiring jobs at different sites in Mexico City.

They do painting jobs in all kinds of buildings. Although painting is their main activity they also carry out a specialised work previous to the painting job like: smoothing and levelling surfaces, plastering walls, etc.

The painters who are "oficiales", or officials the second in the hierarchical organisation of workers in the construction industry, are Zenón and Güicho. Both went to junior high school. They work part time in Mexico City and travel to Houston Tx, USA; for long staying working for an American architect who pays them all their expenses to cross the border and travel to Texas.

With the purpose of comparison another participant in the study was the architect who is the resident at the construction place, he has a five year college degree. David participated in the three studies that follow and conformed the research during the one year and six months that lasted the etnographic working at the natural setting of work.

Procedure

Study 1. Participant observation.

This first approach aims to identify the characteristic features of problem-solving strategies made by adults in everyday mathematical situations within the work context of the group. Also aims to understand the relationship between the collective and individual strategies of mathematical problem resolution and the various ways in which the members organise themselves socially to carry out their tasks.

Study 2. Life histories of the painters and the resident

The purpose of this study is to identify motives, meanings and the sense for the group of the job they carry out. The meaning of the way in which they became masters. Also, to get acquainted with their school histories.

Study 3. Simulation Study

This approach identified problem-solving strategies in an individual form. All pertinent representations were studied among which we can mention: budgets, estimations and measurements.

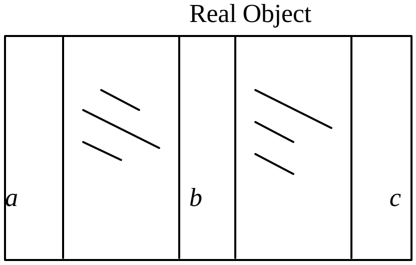
They were given simulation situations based on mathematical tasks that occur naturally. Based on the solution they propose, a dialogue was established in order to have a clear idea of the procedures, strategies and reasons to solve them in a certain way.

The task painters mostly mathematize are: a) to charge for a job by a "tanto" and , b) to charge for the job-week, c) to mix paint, d) to mix gypsum and plaster, d) to mix gypsum sand, e) to estimate, f) to measure, and; g) to budget. The painters create some strategies to solve these tasks mathematicaly:

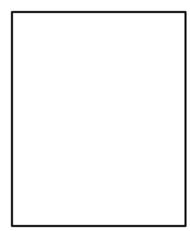
I. Strategies to make work efficient

These strategies have been created since painters started to work and are intended to economise time and physical and intellectual effort. Based upon their experience, the painters do not need to measure each and every wall, ceiling or span to paint. Instead, they visualise together the short distances a, b and c, as an area put altogether, they put together small laps of a wall and compensates this new visualise wall (wall A') with the doors on wall B. In this example walls A and B are in a square angle. The painter doesn't take the measure form wall A, he just takes an even, continuum measure of the length of wall B an then registers the height explaining what he does.

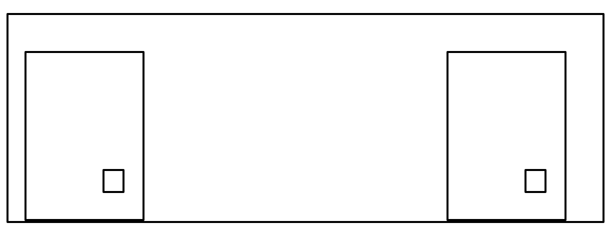
Wall A
Real Object



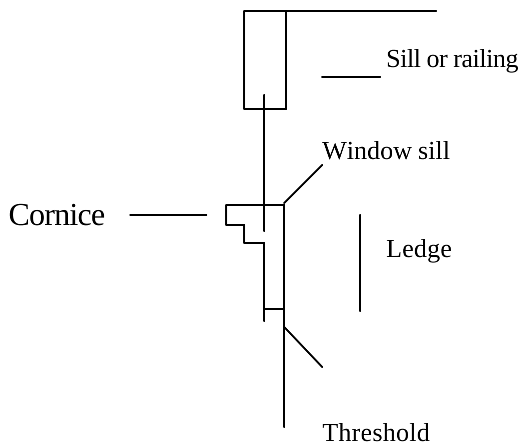
Wall A'
Visualisation



Wall B



The painters develop different strategies to compensate: firsts to balance based on measures from similar systems; second, they create compensations by "tanteo", by trial or by hit like to discount a whole wall "by eye" which has a lot of small laps, and compensate it with a wall nearby of similar form; and third, measuring "by eye" taking for reference the standard measures of what is known as constructive details like window sills, wall brackets, ledges, brick railings, threshold, cornices.



II. Conventional systems to calculate, measure and represent.

The use of unites of the metrical decimal system for the length and the areas as well as volume, the use of unites in the English system like the inch, the gallon, feet and the yard; to convert from the decimal system to the English and vice versa and among measures of the same material.

The visualisation of an arrange in space for a new disposition that will ease and economise cognitive work when measuring, estimating and budgeting. This strategy is used within four different tasks and purposes: 1) in the development of banisters and railings, and grates; 2) in the development of flute or groove doors and gates; 3) in the development of the stairs cylindrical and square spaces; and, 4)at the normalisation of spaces with an irregular shape to a rectangular shape

The use of conventional systems to measure, calculate and represent mostly observed and are evident when they register the data in their notebook shows in the different types of algorithm groupings, like:

1. 3.3 + 4.3 + 3.8 + 3.3 + 2.2 x 1 = 16.9

Instead of:

- [(3.3 x 1) + (4.3 x 1) + (3.8 x 1) + (3.3 x 1) + (2.2 x 1)]

This kind of algorithm grouping among operations shows how they calculate the area. They require to add several lengths and multiply the sum by the wide or by the quantity or number of pieces there is.

2. 1.35 + 3.65 x 2 x .8 = 7.88

Instead of:

- (1.35 + 3.65) 2 x .8
- [1.35 + (3.65 x 2)] x .8
- 1.35 + [(3.65 x 2) (.8)] ...

This grouping uses conventional signs but with ambiguity in the order of the algorithm, this is, the painter don't write which operation and or grouping is necessary to make and in which order for getting the data of the area in square meters.

3. MUROS 4.20 + 2.5 x 2 x 2.53 = 33.9

DESCUENTO 2.95 x 2.53 = 7.46

26.44

Instead of:

- [4.20 + (2.5 x 2) x 2.53] – [2.95 x 2.53] =
- [(4.20 + 2.5) (x 2) x 2.53] – [2.95 x 2.53] =
- [(4.20 + 2.5) (2 x 2.53)] – [2.95 x 2.53] =

The third kind of grouping has to deal with an organised representation of the algorithms, with the use of conventional and non conventional signs (>,   ), and with a higher complexity of calculus and registration.

4. MUROS 5.05 + 3.9 x 2 x 2.23 = 39.91

.25 x 2.53 x 5 = .63   3.16

31.36

DESCUENTOS   2.9 x 2.23 = 6.46

1.55 x 2.23 = 3.45

.85 x 2.23 = 1.89

Instead of:

[(5.05+3.9) (2 x 2.23)] + [(.25 x 2.53) (x5)] – [(2.9 x 2.23) + (1.55 x 2.23) + (.85 x 2.23)] =

[(8.95) (4.46)] + [0.632) (x5)] – [(6.467) + (3.4565) + (1.8955)] =

[39.917] + [3.1625] – [11.819] = 31.2605

This fourth grouping implies a higher level of complexity than the first three. Pablo and Benito use the conventional signs as much as the ones they personally create, this in a clear and exact meaning as well as with ambiguity, also they do multiple inclusive groupings in a sequential mode.

5. *plaf.* 1.48 x (5.64 + 1.34) = 10.33
   7.15 x (2.29 + 2.08 ? 2) = 1.62

- (4.20 + 2.5) x 2 x 2.53 =33.9
- 1.48 x (5.64 + 1.34) = 10.33

The painters write the operation in a horizontal way. Rarely they write in the vertical position the quantities, this vertical way is how we were teach and is the favourite at school. Just twice this position was use: when adding partial quantities for big walls, or different types of spaces, and second when they don't have a calculator and the estimation of the area is difficult to make it mentally. They can do calculations by memory or by writing with five or even six quantities (operators).

To round up is not an strategy they use, not when taking measures not when estimating. Even though they work with decimal and hundreths in a systematic mode.

III. Personal systems to calculate and measure, and to represent

The personal systems to calculate and measure are determine by the activity in a general way, but they are also made according to the specific task in which they are create it, for example to measure by the number of "fists" of lime, or by the exact consistency of the mixture, or taking the measure from the packing, the bottling, the canning or the commercial kind of container of the paint, the lime, the silicon, etc.

They don't make diagrams or design any type of image, drawing, or take or use photographs when they work, take measures, estimate or register their work. All the mediator they write are alphabetical or numeric. They haven't try to use tables, charts or graphs.

It's important to notice that the painter use decimals in every calculate they make, and also the correct use of the zero value. The correct use of the positional value of the zero and the decimals in the writing and calculating.

IV Formal schemes

The formal schemes are the organisers of their mathematical behaviour that we cannot see directly but we infer them based on their repetition property, in the exercise characteristic and in the generalise property of their actions.

With these schemes the painters try to organise the sequences of actions of their intentional de deliberated behaviour. The action and cognitive schemes that we can infer form their every day work when they generalise them the abstract them form the specific context they were construct or create, this is they are no more directly and intimately bond to the tasks but they are still confine to the activity of painting.

In other words, based on this study I cannot say that the painter generalise the formal schemes to other activities beyond painting. Although, there is enough evidence of the functional role some schemes play in different tasks, in different context and scenarios. The main formal schemes observed are:

The theorems in action are propositions held to be true by the subject when she or he acts (Vergnaud, G. 1998:229). Benito's and Pablo's theorems in action are the ones such as: the theorem of the peak roof, the theorem of the cylindrical stair, the theorem of the square cube for the stair.

Other formal schemes were used by the painters in their everyday work that I observed in the natural setting of the construction industry: the proportionality, the use of formulas, the normalisation or rectangularisation of irregular spaces, and the explanations. The two tasks when Pablo and Benito use proportional relations are when they charge for a job and make the lime and paint mixtures. This proportionality relations are direct and generally simple.

V Procedures for control

Five procedures of control are their favourite: the estimations, the control of variables, the visualisation, the order to measure equal to the order to register and to estimate; and verifying all the calculus and the groupings.

The painters, mostly Pablo and Benito, the masters, take the measures and register them in accordance to an order they have establish which is strictly follow and attach to their daily activity of painting. The rules that govern their activity and the ones they have create are: 1) first to measure and register the outside of the house, building or apartment, and then the inside. When measuring and registering the outside first the front, the façade and after the back. 2) First to measure and register the ceilings and after the walls, 3) first to measure and register downstairs and after the first flour and in this order measure and register as many flours as it is, 4) firs to measure an register the length, then the wide and at the end the height.

This order has primarily the function of control among measuring, estimating, budgeting and painting; and it has the purpose of knowing the reference of the measure in the registration while time goes by.

Final notes

The idea of the constitutive role of the environment in practical intellectual activities contrasts with the prevailing conceptions about the relationship between cognition and the world. Cognitive theories based on the computer metaphor see the world as a stage where actors perform the products of their mental operations. For others with a more contextual perspective of the world, the environment is the context, an outer wrapping that greatly affects the cognitive process by means of interpretation processes.

Practical thinking goes beyond the contextualist posture. It points at the clarity a person has of the task to be performed within its environment and to the continual game between the internal representations and operations and the external reality during the problem-solving process. Such game is expressed in the activity theory as the mutual constitution of subject and object. Environment properties are not incorporated to the problem-solving process in a determinant or automatic way, instead they have a functional role only through the deliberate and constructive activities of the person who solves a problem.

Adults do not learn by themselves, the idea of individual and isolated learning is not a natural feature of human beings. On the contrary, adults need a very high degree of specialised skills, abilities, knowledge, means of mediation particularly for the different modes of activities we are involved in. For an adult to learn and work independently and by himself or herself requires a long and slow process of learning that may last many years even decades and that it is only achieved with de collaboration and help of many others on the way: it is a socioculturally phenomena of appropriation and construction of knowledge. Adults are not autodidactique, these is never a lonely job, nevertheless in the classroom, a even never related to development.

References

Aguayo, S. 2000. **El Almanaque Mexicano. Un compendio exhaustivo sobre México en un lenguaje accesible y claro.** México, Grijalbo. Pp. 90-103.

Bishop, A. 1988. "Mathematics Education in its Cultural Context". **Educational Studies in Mathematics.** 19:179-191.

Bueno, Castellanos, Carmen. 1994. **Flor de andamio. Los oficios de la construcción de viviendas en la Ciudad de México.** México, Centro de Investigaciones y Estudios Superiores en Antropología Social. 171p.

Cole, M. and Scribner, Sylvia. 1973. "Cognitive consequences of formal and informal education". **Science**, 183:553-559.

D'Ambrosio, Ubiratan. 1985. "Ethnomathematics and its Place in the History and Pedagogy of Mathematics". **For the learning of mathematics**. 5(1): 44-48.

De la Rocha, Olivia. 1985. "The Reorganisation of Arithmetic Practice in the Kitchen". **Anthropology and Education Quarterly** 16(3): 193-198.

Lave, Jean. 1991. **La condición en la práctica**. México, Paidos, 225p.

Masingila, Joanna, 1994. "Mathematics Practice in Carpet Laying" . **Anthropology and Education Quarterly** 25(4):430-462.

Milroy, Wendy, 1992. "An ethnographic study of the mathematical ideas of a group of carpenters". **Journal fo research in mathematics education.** Monograph number 5.

Saxe, G 1988. "Candy selling and math learning**".** **Educational Research.** 17(6): 14-21.

Scribner, Sylvia. 1997ª . "Knowledge at work". In **Mind and Social Practice. Selected Writings of Sylvia Scribner.** Tobach, Ethel, et al (Eds.). Cambridge University Press,: 308- 318

Scribner, Sylvia. 1997b. "Thinking in Action: Some characteristics of practical thinking". In **Mind and Social Practice. Selected Writings of Sylvia Scribner.** Tobach, Ethel, et al (Eds.) Cambridge University Press, 319-337.

Scribner, Sylvia. 1997c. "Studying work intelligence". In **Mind and Social Practice. Selected Writings of Sylvia Scribner.** Tobach, Ethel et al (Eds.). Cambridge University Press: 338-366.

Scott, P. 1985. "Ethnomathematics: What might it be?" **Newsletter of the International Study Group on Ethnomathematics**. 1(2): 1-2.

Ulloa, M. Y Latapí, P. 1996. ¿En dónde se alfabetizan los mexicanos? Consideraciones y precisiones metodológicas para identificar el origen de la alfabetización entre 1980 y 1990. **Revista Mexicana de Investigación Educativa.** Vol. I, Núm. 1, enero-junio.

Vergnaud, G. 1998. "Towards a Cognitive Theory of Practice". Sierpinska, Anna and Kilpatrick J. (Editors). **Mathematics Education as a Research Domain: A Search for Identity.** Boston, Kluwer Academic Publishers. Pp. 227-240.